BINARY
There are only 10 types of people in this world: Those who understand BINARY and those who don't.
The diagram shows a binary representation of the number $22_{10}$. The switches represent powers of 2, and the switches are turned on for the positions corresponding to the bits in the binary representation of 22.
HOW TO REPRESENT NEGATIVE NUMBERS?

People have tried many things:
1. Using the most significant bit (MSB) as the sign bit: Sign-Magnitude
   • $0101 = +5_{10}$
   • $1101 = -5_{10}$
2. Inverting all bits: 1’s Complement
   • $0101 = +5_{10}$
   • $1010 = -5_{10}$
3. Etc.
4. To explain, we need the concept of modular arithmetic
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Now you try: What is 3+7 MOD 8?

\[
\begin{array}{c}
0 & 1 & 1 \\
+ & 1 & 1 & 1 \\
\hline
1 & 0 & 1 & 0 \\
\end{array}
\]

MOD 8: 0 1 0 = 2_{10}!!

7 is a NEGATIVE NUMBER in MOD8 arithmetic!!!
2’S COMPLEMENT

To Convert a Positive Number into its 2’s Complement (Negative):

1. Invert all bits in the target number
2. Add 1

\[
\begin{array}{c}
\text{2}_10 \\
\downarrow \\
1 \ 0 \ 1 \\
+ \ 1 \\
\hline
\text{-2}_10 \\
1 \ 1 \ 0
\end{array}
\]
MODULAR ARITHMETIC IS LIKE A CLOCK.

2’s Complement works because we are reflecting across the line that separates positive numbers from negative numbers.
2’S COMPLEMENT: EXAMPLE W/ MOD 2^{16} ARITH

Is this number negative or positive?

Negative! Because MSB = 1

One’s: Invert All Bits

2 7 8 1
0010 0111 1000 0001

Two’s: Add 1 to One’s

2 7 8 2
0010 0111 1000 0010

Drop most significant bit because we are doing arithmetic mod 2^{16}
### 2’s Complement: Example w/ Mod 2^{16} Arithmetic

<table>
<thead>
<tr>
<th>1 6 A F</th>
<th>Is this number negative or positive?</th>
<th>Positive! Because MSB = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0001 0110 1010 1111</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**One’s: Invert All Bits**

<table>
<thead>
<tr>
<th>E 9 5 0</th>
<th>1110 1001 0101 0000</th>
</tr>
</thead>
</table>

**Two’s: Add 1 to One’s**

<table>
<thead>
<tr>
<th>E 9 5 1</th>
<th>1110 1001 0101 0001</th>
</tr>
</thead>
</table>

Drop most significant bit because we are doing arithmetic mod 2^{16}